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SOLVING THE STRONG-SHOCK ALGORITHM FOR EXPLOSIVE YIELD AND SPATIAL ORIGIN

by

H. C. Goldwire, Jr.

ABSTRACT

We present a linear least squares solution to the strong-shock algorithm where underground explosive yield and spatial origin are unknown. Also presented are methods for determining standard error estimates for the determined quantities and an illustration of the solution with several sets of simulated hydrodynamic data.

I. INTRODUCTION

The yield of an underground explosion can be determined from measurements of the propagation of the explosion-produced shock wave through the ambient geological medium. For a portion of the shock expansion, the shock radius grows as a power-law function of time. In particular, the shock position is given by

$$\frac{R(t)}{W^{1/3}} = a\left(\frac{t}{W^{1/3}}\right)^b \quad , \tag{1}$$

where time t is measured in milliseconds from explosion time, distance R is in meters from the explosion center, and yield W is in kilotons. Detailed calculations by Eilers, using the 1D F^3 code with realistic equation-of-state data and tuned[1] to reproduce the von Neuman point-source, constant-gamma, analytical solution, showed for tuff and granite that a and b were sensibly constant and were independent of yield.[2] These calculations also provided insight as to the range of applicability of the strong-shock algorithm. Bass and Larsen[3] have performed similar calculations for other media. This algorithm largely forms the basis of the hydrodynamic yield-determination techniques used at the Los Alamos Scientific Laboratory (LASL).

Since spring 1975, we have routinely fielded experiments to determine hydrodynamic yields of LASL nuclear events. Analysis of the data was based on Eq. (1) using the Eilers constants a = 6.29 and b = 0.475, and the results have usually agreed with those obtained from other techniques. We poin out, however, that these experiments were conducted at the Nevada Test Site (NTS) under controlled circumstances: we knew the effective center of the explosion (ECE), i.e., the point of origin of the explosion, and could provide independently determined explosion-time fiducials.

Under less controlled circumstances, the absolute spatial and temporal accuracy of the measuring system may be less than ideal or the ECE may be unknown as, for example, in a verification situation under the Peaceful Nuclear Explosives Treaty (PNET)[4]. Accordingly, we have generalized Eq. (1) to

$$R(t) + R_0 = W^{(1-b)/3}\,(t + t_0)^b \quad . \tag{2}$$

Here, R(t) is the experimentally measured shock-front position at time t, with R and t determined relative to a presumed spatial and temporal origin of the explosion. R_o and t_o are additive corrections to R and t that correct them to the actual explosion time and location. Ideally, experimental R(t) data would be fitted to Eq. (2) to determine any or all of the quantities W, R_o, t_o, a, and b. In practice, a and b are usually assumed known, and the combinations of unknowns we most commonly expect to encounter are (1) W, R_o, (2) W, t_o, or (3) W, R_o, t_o.

It is the purpose of this report to present a linear least squares solution to the yield and R-shift (W, R_o) problem and to illustrate its use with several examples.

II. ANALYSIS

For this problem, we assume that a, b, and t_o are known and rewrite Eq. (2) as

$$R(t) = c\, x_1(t) + d\, x_2(t) \quad , \tag{3}$$

where

$$c = a\, W^{(1-b)/3}, \quad d = -R_o \quad , \tag{4}$$

$$x_1(t) = (t + t_o)^b, \quad x_2(t) \equiv 1 \quad . \tag{5}$$

Equation (3) can be solved by linear least squares regression for the desired constants c and d and for the standard error estimates σ_c, σ_d, and covariance σ_{cd}. Given the data set (t_i, R_i, σ_i; i = 1, N), where σ_i is the statistical uncertainty to be associated with the value R_i, we define the auxuliary sums

$$A = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \qquad D = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} R_i\, x_1(t_i)$$

$$B = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} x_1(t_i) \qquad E = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} R_i \tag{6}$$

$$C = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} x_1^2(t_i) \qquad F = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} R_i^2 \quad .$$

Then the desired least square quantities and the corresponding uncertainties are

$$c = (DA - BE)/\Delta \ , \ d = (CE - BD)/\Delta \tag{7}$$

and

$$\sigma_c^2 = A/\Delta \ , \ \sigma_d^2 = C/\Delta \ , \ \sigma_{cd} = -B/\Delta \ , \tag{8}$$

where

$$\Delta = (AC - B^2) \quad .$$

In terms of these quantities, our original quantities W and R_o and their formal uncertainties then are given by

$$W = \left(\frac{c}{a}\right)^{3/(1-b)} \qquad \sigma_W = W\sqrt{A/\Delta} \Big/ c\left(\frac{1-b}{3}\right)$$

$$R_o = -d \qquad \sigma_{R_o} = \sqrt{C/\Delta} \quad . \tag{9}$$

If the individual standard deviations σ_i are unknown or if an unweighted fit is desired, the σ_i in Eqs. (6) should all be set equal to a constant σ_o (to be determined). Note that in this case σ_o will cancel out of Eqs. (7), allowing c and d to still be determined. For Eqs. (8), however, we can obtain an unbiased statistical estimate for σ_o from σ_R, the standard deviation of the data about the fit. In particular, we calculate σ_R from

$$\sigma_R = \left\{ \frac{\sum_{i=1}^{N} [R_i + R_o - a\, W^{(1-b)/3} (t_i + t_o)^b]^2}{N - 2} \right\}^{1/2} \quad . \tag{10}$$

or with less precision from the auxiliary sums

$$\sigma_R = \left\{ \frac{c^2 C + 2cdB - 2cD + Ad^2 - 2dE + F}{N - 2} \right\}^{1/2} \quad . \tag{11}$$

III. TWO EXAMPLES

To illustrate this least squares method, we present in Tables I and II two sets of simulated hydrodynamic data. The labels for the quantities in these tables are explained in Table III.

A. Properties of the Generated Data Sets

Using Eq. (1) with a yield of 150 kt, data were generated at 100-μs intervals over the time span 1.0-3.5 ms, the approximate range normally analyzed for such a yield. These algorithmic data were then modified by adding 5.000 m to all points (thereby simulating the effects of an origin shift or an absolute calibration error) and by adding random-noise deviations to simulate the effects of noisy data. The noise levels chosen, rms deviations of 4.1 and 5.3 cm per point, correspond to high-quality data, but such levels are achievable today. For a medium sonic velocity of 3.0 m/ms, the data are all presonic and hence usable. (The sonic time and radius would be 5.27 ms and 33.30 m, respectively.)

B. Results of the Least Squares Fits

Tables I and II illustrate calculation results at added noise levels of 4.1 and 5.3 cm, respectively. The least squares solutions agree very well with the "correct" answer WALG = 150 kt and RSHIFT = −5.00 m. Also, the formal ranges of uncertainty for the two determined quantities, WFIT ± SIGW and RSHIFT ± RSIGR0, do encompass the correct answer. Work is in progress on a statistical analysis of man such examples as are presented in these tables.

It should be pointed out that analyses of actual hydrodynamic data will not, in general, be so successful. Among the reasons for this are the following.

1. Less data may be obtained.
2. Noise sources may not be strictly Gaussian.
3. The algorithmic region of data may be restricted or difficult to identify.
4. The algorithm is only an approximation to actual physics of expansion.
5. Explosions may not be point sources.

IV. CONCLUSIONS

This least squares method enables one to efficiently and effectively solve Eq. (2) for R_o and W, assuming that t_o, a, and b are known. This method was shown to work successfully for the simulated data of Tables I and II. A number of statistical quantities of interest were also calculated and are presented in the tables. To the extent that data noise sources are Gaussian and the data follow the strong-shock algorithm, this least squares method is statistically the most powerful and appropriate technique to use for solving for yield and shifts of origin.

REFERENCES

1. D. D. Eilers, "A Numerical Integration of a 97 kt Explosion in Sea Level Air," Los Alamos Scientific Laboratory report LAMS-2985 (December 1963).

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3. R. C. Bass and G. E. Larsen, "Shock Propagation in Several Geologic Materials of Interest in Hydrodynamic Yield Determinations," Sandia Laboratories, Albuquerque, report SAND 77-0402 (March 1977).

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TABLE I

LINEAR LEAST SQUARES TEST CASE
(noise level ≈ 4.1 cm per point)

PROPERTIES OF GENERATED DATA SET

NPTS=	26	WALG=	150.00	TS=	5.27	TADD=	0.000	TSTART=	1.00	NOISE SIGMA=	.0411
		VS=	3.00	RS=	33.30	RADD=	5.000	TSTOP=	3.50	NOISE MEAN=	.0002

PROPERTIES OF LEAST SQUARES FIT TO DATA — CSAB= .05701

WFIT=	149.5408	RSHIFT=	-5.0119	SIGR=	.041890	CSR=	.04179	AFIT=	6.28663
SIGW=	1.9313	RSIGRO=	.0501	RATIO=	1.019424	FACT=	.99754	SIGA=	.01421

POINT	TIME	RALG+SM	RDATA	RFIT	DELR	NOISE	WCALC	W-WALG
1	1.00	20.1170	20.1318	20.1209	.0109	.0147	150.1588	.1588
2	1.10	20.8171	20.8166	20.8206	-.0040	-.0005	149.3268	-.6732
3	1.20	21.4846	21.5138	21.4877	.0261	.0292	150.9011	.9011
4	1.30	22.1234	22.1247	22.1262	-.0015	.0013	149.4676	-.5324
5	1.40	22.7369	22.7172	22.7393	-.0221	-.0197	148.4771	-1.5229
6	1.50	23.3278	23.3880	23.3299	.0581	.0602	152.2707	2.2707
7	1.60	23.8983	23.8358	23.9001	-.0644	-.0625	146.6526	-3.3474
8	1.70	24.4505	24.4913	24.4520	.0393	.0408	151.2763	1.2763
9	1.80	24.9858	24.9386	24.9870	-.0484	-.0472	147.4823	-2.5177
10	1.90	25.5057	25.4982	25.5066	-.0084	-.0075	149.1890	-.8110
11	2.00	26.0114	25.9997	26.0121	-.0124	-.0118	149.0355	-.9645
12	2.10	26.5041	26.5312	26.5045	.0267	.0271	150.6065	.6065
13	2.20	26.9845	26.9595	26.9847	-.0251	-.0250	148.5655	-1.4345
14	2.30	27.4537	27.5221	27.4536	.0685	.0685	152.1698	2.1698
15	2.40	27.9122	27.8781	27.9119	-.0337	-.0341	148.2863	-1.7137
16	2.50	28.3608	28.3405	28.3602	-.0197	-.0203	148.8216	-1.1784
17	2.60	28.8001	28.7019	28.7993	-.0974	-.0983	146.0742	-3.9258
18	2.70	29.2306	29.3076	29.2296	.0780	.0770	152.3152	2.3152
19	2.80	29.6528	29.6560	29.6515	.0045	.0032	149.6961	-.3039
20	2.90	30.0672	30.0780	30.0657	.0123	.0108	149.9605	-.0395
21	3.00	30.4741	30.4865	30.4724	.0141	.0124	150.0156	.0156
22	3.10	30.8740	30.8246	30.8721	-.0475	-.0494	147.9787	-2.0213
23	3.20	31.2671	31.2634	31.2650	-.0016	-.0037	149.4903	-.5097
24	3.30	31.6539	31.6850	31.6515	.0335	.0311	150.6170	.6170
25	3.40	32.0345	32.0721	32.0320	.0401	.0375	150.8126	.8126
26	3.50	32.4094	32.3807	32.4066	-.0259	-.0287	148.7343	-1.2657
				MEANS	-.0000	.0002	149.5532	-.4468
				SIGMA	.0419	.0411	1.6321	1.6321

AUXILIARY QUANTITIES

CC=	15.10893	SX =	3.7648762E+01
SC=	.03415	SX2=	5.6021318E+01
DD=	5.01195	SRX=	1.0351156E+03
SD=	.05013	SR =	6.9914299E+02
SCD=	-.96227	SR2=	1.9143594E+04

TABLE II

LINEAR LEAST SQUARES TEST CASE
(noise level ≈ 5.3 cm per point)

PROPERTIES OF GENERATED DATA SET

NPTS=	26	WALG=	150.00	TS=	5.27	TADD=	0.000	TSTART=	1.00	NOISE SIGMA=	.0534
		VS=	3.00	RS=	33.30	RADD=	5.000	TSTOP=	3.50	NOISE MEAN=	.0047

PROPERTIES OF LEAST SQUARES FIT TO DATA

CSAB= .36876

WFIT=	148.9673	RSHIFT=	-5.0312	SIGR=	.054268	CSR=	.05474	AFIT=	6.28240
SIGW=	2.4941	RSIGRO=	.0649	RATIO=	1.017015	FACT=	1.00878	SIGA=	.01841

POINT	TIME	RALG+5M	RDATA	RFIT	DELR	NOISE	WCALC	W-WALG
1	1.00	20.1170	20.1877	20.1300	.0577	.0706	152.2501	2.2501
2	1.10	20.8171	20.8596	20.8292	.0303	.0424	150.6090	.6090
3	1.20	21.4846	21.4600	21.4958	-.0358	-.0245	147.1249	-2.8751
4	1.30	22.1234	22.1322	22.1339	-.0017	.0085	148.8850	-1.1150
5	1.40	22.7369	22.7321	22.7466	-.0145	-.0048	148.2706	-1.7294
6	1.50	23.3278	23.3159	23.3368	-.0210	-.0119	147.9951	-2.0049
7	1.60	23.8983	23.8081	23.9067	-.0986	-.0903	144.5741	-5.4259
8	1.70	24.4505	24.4871	24.4581	.0289	.0366	150.2394	.2394
9	1.80	24.9858	24.9901	24.9928	-.0027	.0044	148.8535	-1.1465
10	1.90	25.5057	25.5268	25.5121	.0147	.0211	149.5774	-.4226
11	2.00	26.0114	25.9251	26.0172	-.0921	-.0863	145.2690	-4.7310
12	2.10	26.5041	26.5447	26.5093	.0354	.0406	150.3754	.3754
13	2.20	26.9845	27.0232	26.9892	.0340	.0386	150.2902	.2902
14	2.30	27.4537	27.5346	27.4577	.0768	.0809	151.9070	1.9070
15	2.40	27.9122	27.8967	27.9157	-.0190	-.0155	148.2626	-1.7374
16	2.50	28.3608	28.4044	28.3638	.0406	.0435	150.4541	.4541
17	2.60	28.8001	28.8796	28.8025	.0771	.0795	151.7497	1.7497
18	2.70	29.2306	29.1265	29.2325	-.1060	-.1041	145.2778	-4.7222
19	2.80	29.6528	29.7250	29.6542	.0708	.0722	151.4307	1.4307
20	2.90	30.0672	30.0289	30.0681	-.0392	-.0383	147.6394	-2.3606
21	3.00	30.4741	30.4934	30.4745	.0188	.0192	149.5986	-.4014
22	3.10	30.8740	30.8478	30.8739	-.0261	-.0262	148.1088	-1.8912
23	3.20	31.2671	31.18[illegible]4	31.2666	-.0852	-.0857	146.2254	-3.7746
24	3.30	31.6539	31.6406	31.6529	-.0123	-.0133	148.5747	-1.4253
25	3.40	32.0345	32.0541	32.0331	.0211	.0196	149.6324	-.3676
26	3.50	32.4094	32.4554	32.4074	.0479	.0460	150.4634	.4634
				MEANS	-.0000	.0047	148.9861	-1.0139
				SIGMA	.0543	.0534	2.1314	2.1314

AUXILIARY QUANTITIES

CC=	15.09877	SX =	3.7648762E+01
SC=	.04424	SX2=	5.6021318E+01
DD=	5.03119	SRX=	1.0352710E+03
SD=	.06494	SR =	6.9926087E+02
SCD=	-.96227	SR2=	1.9149501E+04

TABLE III

DEFINITIONS

Label	Explanation
NPTS	Number of generated algorithm points
WALG	Algorithmic yield
VS	Sonic velocity of medium
TS,RS	Sonic time and radius
TADD,RADD	Time and radius increments added to algorithmic data
TSTART,TSTOP	Time span of data
NOISE SIGMA	Standard deviation of random noise deviates
NOISE MEAN	Mean of deviations
WFIT	Least squares fitted value of yield W
SIGW	σ_W
RSHIFT	Least squares fitted value of R_o
RSIGR0	σ_{R_O}
SIGR	σ_R
RATIO	σ_R/noise sigma
CSR	An "approximation" to σ_R
FACT	CSR/σ_R
AFIT	Least squares fitted value of a, assuming W fixed at value WALG
ASIG	σ_a
RALG + 5M	Algorithmic data + 5:000 m
RDATA	Data analyzed = RALG + 5M + NOISE
RFIT	Resulting fit to data
DELR	Deviations, RDATA - RFIT
NOISE	Noise deviates added to algorithmic data
WCALC	Calculated yields for individual data points corresponding to fitted values of W and R_o
W-WALG	WCALC - WALG

Unlabeled quantities below columns labeled DELR, NOISE, WCALC, and W-WALG in Tables I and II are means and standard deviations of entries in the corresponding columns.

CC	c	
SC	σ_c	
DD	d	
SD	σ_d	
SCD	σ_{cd}	
SX	B	Multiplied by σ_R^2
SX2	C	Multiplied by σ_R^2
SRX	D	Multiplied by σ_R^2
SR	E	Multiplied by σ_R^2
SR2	F	Multiplied by σ_R^2